

Problem 1.40

[Difficulty: 2]

1.40 Calculate the density of standard air in a laboratory from the ideal gas equation of state. Estimate the experimental uncertainty in the air density calculated for standard conditions (29.9 in. of mercury and 59°F) if the uncertainty in measuring the barometer height is ± 0.1 in. of mercury and the uncertainty in measuring temperature is $\pm 0.5^\circ\text{F}$. (Note that 29.9 in. of mercury corresponds to 14.7 psia.)

Given: Air at standard conditions – $p = 29.9$ in Hg, $T = 59^\circ\text{F}$

Uncertainty in p is ± 0.1 in Hg, in T is $\pm 0.5^\circ\text{F}$

Note that 29.9 in Hg corresponds to 14.7 psia

Find: Air density using ideal gas equation of state; Estimate of uncertainty in calculated value.

Solution:

$$\rho = \frac{p}{RT} = 14.7 \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lb} \cdot ^\circ\text{R}}{53.3 \text{ ft} \cdot \text{lbf}} \times \frac{1}{519^\circ\text{R}} \times 144 \frac{\text{in}^2}{\text{ft}^2}$$

The uncertainty in density is given by

$$u_\rho = \left[\left(\frac{p}{\rho} \frac{\partial \rho}{\partial p} u_p \right)^2 + \left(\frac{T}{\rho} \frac{\partial \rho}{\partial T} u_T \right)^2 \right]^{\frac{1}{2}}$$

$$\frac{p}{\rho} \frac{\partial \rho}{\partial p} = RT \frac{1}{RT} = \frac{RT}{RT} = 1; \quad u_p = \frac{\pm 0.1}{29.9} = \pm 0.334\%$$

$$\frac{T}{\rho} \frac{\partial \rho}{\partial T} = \frac{T}{\rho} \cdot -\frac{p}{RT^2} = -\frac{p}{\rho RT} = -1; \quad u_T = \frac{\pm 0.5}{460 + 59} = \pm 0.0963\%$$

Then

$$u_\rho = \left[u_p^2 + (-u_T)^2 \right]^{\frac{1}{2}} = \pm \left[0.334\%^2 + (-0.0963\%)^2 \right]^{\frac{1}{2}}$$

$$u_\rho = \pm 0.348\% = \pm 2.66 \times 10^{-4} \frac{\text{lbm}}{\text{ft}^3}$$